EVALUATING SPACE COLLISION SCENARIOS WITH CLOSEAP

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Abstract

In recent years, the space debris has gained a lot of interest as part of the space environment due to the increasing population of uncontrolled man-made objects orbiting the Earth, which is causing a significant risk of collision with the operating satellites. Currently, there are more than 600,000 objects larger than 1 cm in orbit (according to the ESA MASTER-2005 model) particularly concentrated in the most busy and interesting orbits for current and future missions; and after each collision or event a considerable amount of new debris appear in space.

CRASS for Collision Risk Assessment and ODIN for Orbit Determination have been developed as part of the initiative for a future European Space Situational Awareness System to answer the concerns of satellite operators. These software packages initially used by the Space Debris Office at ESOC are also the basis for closeap, GMV’s integrated solution for the analysis and mitigation of space collision situations. Implementation of the CRASS algorithms of Smart Sieve and revised and up to date SGP equations in the orbit propagation module have led to a more efficient assessment of the conjunctions, to predict the collisions and provide this as input for the computation of avoidance manoeuvres. The revised SGP models have improved the deficiencies of the previous implementations and have been integrated with NAPEOS (Navigation Package for Earth Orbiting Satellites) to improve the orbit determination, prediction and analysis capabilities.

In this work the implementation of these algorithms in closeap, and the resulting capabilities for conjunction assessment, and collision probability prediction are described. Whereas the algorithms are normally described from a general point of view and also analyzed in this way, the interesting situations are those not covered by the generic approach. All software used in closeap is operational and fully validated; however, the nature of the collision risk assessment problem leaves always a certain level of uncertainty about whether all possible approach events have been detected. The work described by this paper focuses in the identification of those limiting scenarios and the ability of closeap to detect them and to provide adequate collision probability values that can be further used operationally.

Symbols

- \( R_{acc} \): Acceleration expanded critical volume
- \( \vec{\rho} \): Relative position vector
- \( \vec{\dot{\rho}} \): Relative velocity of target and chase
- \( P_C \): Probability of Collision
- \( \vec{x}_t \): Target satellite position in J2000
- \( \vec{v}_t \): Target satellite velocity vector in J2000
- \( \vec{v}_c \): Chaser velocity vector in J2000
- \( \vec{x}_c \): Chaser position vector in J2000
- \( F \): Sum of the chaser and target covariance matrices
INTRODUCTION

The ever increasing population of space debris poses a major threat to operational satellites. There are 2300 non operational satellites in orbit, and more than 900 active satellites. Unfortunately, this has been improved recently by the collision between the operational satellite Iridium-33 and the already decommissioned satellite Cosmos-2251 on February 10th, 2009. This type of events are not only a serious problem for the owner of the satellite, but also for the rest of the satellite operators since more space debris is generated due to the collision in specially interesting orbital slots. As there is currently no effective way of removing debris from orbit, the best approach is to control the production of debris. Thus, it is important to have reliable tools for collision risk assessment and avoidance manoeuvre optimization.

To assess the collision risk, different systems have been developed, and different collision probability estimation models have been suggested. The estimations significantly depend on the orbit determination accuracies, and dramatically change with the geometry of conjunction. The Iridium and Cosmos collision showed that these systems still can not predict all the close conjunctions.

The main focus of this work is investigating these limiting scenarios for the model used in closeap.

As part of GMV’s activities in this field, GMV has developed closeap, a tool for conjunction assessment, collision probability prediction and collision avoidance manoeuvre optimization integrated in focusSuite, GMV’s commercial off-the-shelf (COTS), multi-mission, multi-satellite flight dynamics solution for satellite control. Main closeap features are:

- Catalogue filtering, conjunction assessment and collision risk algorithms inherited from CRASS
- Orbit propagation library and computational core from NAPEOS
- the latest implementation of the Simplified General Perturbations (SGP) theory to compute space debris orbits based on publicly available Two Line Elements (TLE)
- Full integration within focusSuite
- Optimization of collision avoidance manoeuvres in case of high risk

closeap makes use of the SGP theory to propagate those TLE sets in a very efficient way. This precise orbit propagator is used to propagate with high accuracy the orbit of the operational satellites under consideration and to propagate the state covariance of both the chaser object and the target satellite. In the closeap, the corrected version of SDP4 along with the GMV fitting method, are used to increase the accuracy of orbit propagation of the TLEs. The fitting method used in closeap has improved the performance of the SDP4 algorithm significantly.

Catalogue filtering techniques

Due to the large amount of space debris in space, the propagation of all objects along a period of time on the order of several days is a computationally intensive task. In order to accelerate this process, those objects whose orbital properties make it impossible for them to collide with the target satellite are filtered out.

Figure 1 - closeap inheritance scheme

The Smart Sieve algorithm used in closeap includes series of filters based on the relative position and velocity of target and object. It has been designed in a way to minimize the computational cost of the conjunction identification process.
The filters included in the Smart Sieve are:

- **Apogee-Perigee filter** using the apogee and perigee radius of both target and chaser, discards any pair where the chaser altitude interval doesn’t intersect the target satellite altitude belt.

- **Sieve techniques** that compute the distance between target and chaser at specific time steps; if this distance is larger than a prescribed threshold the pair is discarded. Different sieves with different distance threshold are used in closeap. The threshold distance is increased with increasing step size. The filters used in closeap Smart Sieve method are:

  - **X, Y, Z sieve**: using the escape velocity, if the relative distance in any of the X, Y, and Z direction is more than the escape velocity threshold volume, the pair is more than this threshold apart.

  - **Range squared sieve**: Using the same threshold distance as the previous filter, but comparing it to the distance between target and chaser.

  - **Minimum distance sieve**, this filter discard subjects for which:

    \[ \rho_{\text{min}}^2 = \rho^2 - (\tilde{\rho} \cdot \tilde{\rho})^2 > R^2_{\text{acc}} \quad (1) \]

    Taking in to account the curvature:

    \[ R_{\text{acc}} = R_{\text{cr}} + g_o \Delta t^2 \quad (2) \]

  - **Refined range squared sieve**: The refined threshold distance in these filters uses the relative speed of target and chaser, along with taking into account the curvature effects:

    \[ R_{\text{th,ref}} = R_{\text{acc}} + \frac{1}{2} \left[ \frac{1}{\rho_0} \cdot \frac{\tilde{\rho}_0 \cdot \tilde{\rho}}{|\tilde{\rho}| \cdot |\tilde{\rho}|} \right] \Delta t \quad (3) \]

  - **Fine conjunction detection**: All the pairs that pass the above filters then pass a numerical root finder that accurately predicts the time of closest approach. The root finder finds the zero (minimum) value of $\tilde{\rho} \cdot \tilde{\rho}$. In the CRASS algorithm implementation, the optimum step size for the root finder is found to be 180 second, for comparing all the objects in USSPACECOM database. But the optimum value depends on the size of database and number of targets.

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**Collision Risk Assessment in closeap-CRASS**

The Collision Risk Assessment algorithm of closeap is based on the previous software of the European Space Agency, CRASS. To choose between different methods that have been suggested for Risk Assessment up to this time, different analysis and simulations were done in CRASS development process. The finally method suggested by Alfriend and Akella was accredited to be used in CRASS. The assumptions of using this method are:

- Rectilinear motion of both target and chaser, a justified assumption for short encounter duration.

- No uncertainty in the velocity and constant position uncertainty.

- Gaussian position uncertainty distribution. The position probability density function is defined as:

  \[ p(\Delta \tilde{r}) = \frac{1}{\sqrt{(2\pi)^3 \det(F)}} \exp \left[ -\frac{1}{2} \Delta \tilde{r}^T F^{-1} \Delta \tilde{r} \right] \quad (4) \]

- The method that is used specifically in closeap adds another assumption to the above conditions when scarce information
is available for the debris object that are common among the regular methods used to estimate the probability of collision. Alfriend and Akella assume the debris object can be considered as a point mass, as a result of the relative size of target and debris.

Assuming $F$ to be the sum of the position covariance matrices of target and chaser, the probability of collision is then defined as the probability that the debris object will intercept the sphere of radius $R$ during the encounter:

$$P_c = \frac{1}{\sqrt{(2\pi)^{n}det(F)}} \int_V \exp \left[ -\frac{1}{2} (\hat{\beta} - \hat{\rho}_0)^T F^{-1} (\hat{\beta} - \hat{\rho}_0) \right] dV$$  \hspace{1cm} (5)

With the first assumption, the problem can be reduced to the plane perpendicular to the relative velocity vector, containing both objects at the time of closest approach. This plane is defined by $i$ and $j$ unit vectors in the following coordinate system:

$$\vec{i} = \frac{\vec{v}_r}{v_r}, \quad \vec{j} = \frac{\vec{v}_0}{v_0}, \quad \vec{k} = \vec{i} \times \vec{j}$$

Using the conjunction plane definition and projecting the problem to this plane, which is acceptable with the rectilinear motion assumption described previously, the integral can be reduced to a plane integral rather than a volume integral:

$$P_c = \frac{1}{\sqrt{(2\pi)^{n}det(F)}} \int_{-R}^{R} \int_{-\pi/2}^{\pi/2} \exp \left[ -\frac{1}{2} (\hat{\beta} - \hat{\rho}_0)^T \Sigma P^* (\hat{\beta} - \hat{\rho}_0) \right] d\beta d\alpha$$ \hspace{1cm} (6)

Where $P^*$ and $\hat{s}$ are covariance matrix and miss vector, mapped to the conjunction plane.

$$\hat{s} = x\hat{i} + z\hat{k}, \quad \hat{s}_0 = x_0\hat{i} + z_0\hat{k}$$

This reduced form is obtained by Akella and Foster.

**METHODOLOGY**

The main objective of this paper is to assess different collision scenarios using closeap, to examine whether it properly determines the collisions, and if it can identify all conjunctions. There is always a certain level of uncertainty about whether all possible conjunctions are detected or not. The objective of the current study is to further investigate these limiting scenarios, and the ability of closeap in detecting them. The limiting scenarios can be due to the limitations in filtering process, or an underestimated collision probability; the latter is the main focus of this work.

There are different parameters involved in probability computation. Generally, the factors that influence the probability of collision are:

- Covariance size
- Correlation matrix
- Orientation of target orbit with respect to J2000 (ascending node and argument of perigee)
- Geometry of conjunction
- Operational Parameters, associated with the numerical methods used, e.g. Smart Sieve Filter time steps and Accuracy of close Approach finder

Here in this work, the efforts are aimed at analyzing the change of probability for different collision geometries. The geometries that lead to lower probability levels are considered critical, and the process may not detect them with a change in covariance matrix or slight changes in miss distance. Estimates of the probability for different orbits, and for different conjunction geometries, are examined to find the limiting scenarios.

**Target Centred Coordinate System**

The coordinate system of Radial, Cross-track and Along-track directions is used to define the relative position and velocity of target and chaser. Using the position and velocity vector of target satellite in the J2000 Earth centred coordinate system, the unit vectors of radial, along track and cross track are defined as:
Figure 4 - Target Centred Coordinate System: Radial, Along-Track and Cross-Track

**Conjunction Geometry Definition**

To define different conjunction geometries, 6 parameters have been included: (see Figure 5)

1. **Range** (R): Modulus of the relative position vector of target and chaser (miss vector) in target centred orbit coordinate system. The miss vector in the Geocentric coordinate system can be converted to the target centred coordinate system by using the transformation matrix that is defined in section 3-1:

\[
\vec{u} = \frac{\vec{x}_t}{|\vec{x}_t|}
\]

\[
\vec{w} = \frac{\vec{x}_t \times \vec{v}_t}{|\vec{x}_t \times \vec{v}_t|}
\]

\[
\vec{v} = \frac{\vec{w} \times \vec{u}}{|\vec{w} \times \vec{u}|}
\]

\[
C = [\vec{u} \quad \vec{v} \quad \vec{w}]
\]

2. **Range Rate** (\(\dot{R}\)): Modulus of the Relative velocity vector in a target centred orbit coordinate system

\[
\vec{x}_c^\prime = C^T(\vec{x}_t^\prime - \vec{x}_c^\prime)
\]

\[
R = \left| \vec{x}_c^\prime \right|
\]

3. **Azimuth Angle** (A): measured as a right handed rotation about the radial direction

\[
A = \arctan \frac{\vec{x}_{c2}}{\vec{x}_{c1}}
\]

Azimuth angle is defined between -180 and 180 degrees.

4. **Elevation Angle** (h): Angle between the miss vector and local horizontal plane, which is defined positive when moving toward deep space

\[
h = \arcsin \frac{\vec{x}_{c1}}{R}
\]

Approach velocity can be expressed in the same way as the miss vector, in terms of Range Rate described earlier, Approach azimuth and Approach Elevation.

5. **Approach Azimuth** (A_{App}):

The same way that Azimuth angle is defined, approach azimuth is defined, using the approach velocity vector

\[
A_{App} = \arctan \frac{v_{c1}}{v_{c2}}
\]

6. **Approach Elevation** (h_{App}):

The same way that Elevation angle is defined, approach Elevation is defined, using the approach velocity vector

\[
h_{App} = \arcsin \frac{v_{c1}}{R}
\]
Assessing the collision prediction with closeap

Several tests are defined to examine the collision probability change with the geometry of conjunction. The range, elevation, azimuth, and range-rate, approach elevation and approach azimuth are used to define the scenarios and the chaser state vector is derived from these values, as explained in the following lines:

\[
x_a(1) = |R| \times \sin (h) \\
x_a(2) = \frac{|R| \times \cos (h)}{\sqrt{1 + \tan(A)^2}} \\
x_a(3) = \frac{|R| \times \cos (h) \times \tan (A)}{\sqrt{1 + \tan(A)^2}}
\]

\[
v_a(1) = |\text{Range Rate}| \times \sin (h_{\text{App}}) \\
v_a(2) = \frac{|\text{Range Rate}| \times \cos (h_{\text{App}})}{\sqrt{1 + \tan(A_{\text{App}})^2}} \\
v_a(3) = \frac{|\text{Range Rate}| \times \cos (h_{\text{App}}) \times \tan (A_{\text{App}})}{\sqrt{1 + \tan(A_{\text{App}})^2}}
\]

The covariance of objects (chasers) is extracted from the table based on the inclination, semi major axis and eccentricity of its orbit at the epoch. The sigma values can change using the coefficients in closeap configuration (tables 1 and 2). These coefficients are used later in the tests to examine the effect of covariance on probability.

For the operational satellites, the ground control centre that tracks the satellite should have the covariances at epoch. In orbit propagation, the parameter errors change depending on the propagation arc length. In practice here, the operational values of the propagated position error, for ERS-2 satellite, is used as satellite covariance matrix. The epoch position errors are given in table 3.

And the correlation matrix used for target is defined by propagating a unity correlation matrix for two days. The resulting correlation matrix is

<table>
<thead>
<tr>
<th>U (Radial) (km)</th>
<th>0,01</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (Along Track) (km)</td>
<td>0,1</td>
</tr>
<tr>
<td>W (Cross Track) (km)</td>
<td>0,01</td>
</tr>
<tr>
<td>Vu (km/s)</td>
<td>0,0001</td>
</tr>
<tr>
<td>Vv (km/s)</td>
<td>0,000001</td>
</tr>
<tr>
<td>Vw (km/s)</td>
<td>0,000001</td>
</tr>
</tbody>
</table>

Table 3 - State vector error for target satellites

TEST RESULTS

The tests are first done to examine the performance of system to assess close conjunction in different orbits. The orbits are presented in table 4.

Test results for a target in LEO are shown in figures 6, 7 and 8, for different approach angles. The miss distance in these set of tests is set to zero, and range rate is kept constant.

The same set of tests is done with a target in GEO. Figure 9 shows the resulting graph. It is important to remark that the graphs for LEO and GEO have significant differences when the approach elevation angle is not zero.

The probability of collision decreases when the distance between two objects is increased. But the change is different for different geometries. Figure 11 shows three different geometries, the details of which are given in the table 5.

The probability reaches its maximum when the approach direction is in the direction of miss distance. Elevation and approach elevation are set as equal and the peak happens when the approach azimuth is equal to the azimuth angle, e.g. when the approach direction coincides with the miss distance direction. This is shown in Figure 12, in which two different miss distance directions of tests 3 and 4 of table 5, result in a different maximum probability approach direction.

The probability of collision for test 2 of table 5 is less than the threshold probability. The
probability change with miss distance is shown in the next graph, figure 13. The probability falls below the threshold limit very quickly.

**Effect of covariance size**

Using the first geometry, and the LEO target state, the change of probability levels with covariance is studied. Figure 14 shows how a change in debris covariance affects the collision probability levels. The covariance factor is set to 4 different values. The probability levels decrease significantly with increase in covariance factor.

**Limitations**

Although in most of the tests, with few exceptions, the probability of collision was above the threshold probability that is used in operational purposes, it can still decrease below this threshold with changes in factors such as state vector error covariance, and changes in miss distance, *Smart Sieve* Filter time steps and accuracy of close approach finder.

An increase in miss distance decreases the collision probability. Also, as is shown in test case c (figure 11), the decrease in probability is rapid in some test cases. The change is faster if the miss distance is out of the collision plane.

While threshold distance is set to 10 kilometres, a 0.5 kilometre miss distance between two objects should return a probability higher than the 0.0000001 limit. But the results of test case 2 in graph were clearly less than the threshold probability.

**Covariance Size:** The accuracy of TLE objects is low, and the probability falls with the increase in covariance. The accuracy of orbit determination and accuracy of TLE data, with the prediction arc length affects the covariance size. Propagating the state vector sigma values for two days leads to an order of magnitude increase in the state vector sigma values and a significant increase in covariance. Keeping this in mind, the probability again may fall under the threshold limit, providing the covariance increases enough.

Using the general trend of collision probability, for example, in LEO, as seen in the graphs, the minimum probabilities are estimated for central collision, when approach angle is around 90 or -90 degrees. If in these angles the covariance changes, at some point the method will not estimate a high enough probability to report the conjunction.

Although tests done in CNES [5] and tests done by *closeap* [1] showed that the Iridium-Cosmos collision of February 2009 was a predictable conjunction, it was not reported by the operators of Iridium and Pentagon. Some of these collisions were tested with *closeap* previously and the results are presented in [1].

The table below shows the probability of collision for these collisions:

<table>
<thead>
<tr>
<th>Conjunction event</th>
<th>Miss distance (km)</th>
<th>Collision probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENVISAT vs. COSMOS 1269</td>
<td>1.38</td>
<td>1.66e-05</td>
</tr>
<tr>
<td>ENVISAT vs. ZENITH-2</td>
<td>0.26</td>
<td>1.66e-04</td>
</tr>
<tr>
<td>ENVISAT* vs. ZENITH-2</td>
<td>0.38</td>
<td>3.48e-05</td>
</tr>
<tr>
<td>ERS-2 vs. COSMOS-3M</td>
<td>0.14</td>
<td>1.07e-04</td>
</tr>
<tr>
<td>IRIDIUM-33 vs. COSMOS-2251</td>
<td>0.70</td>
<td>1.64e-05</td>
</tr>
</tbody>
</table>

(* with true collision avoidance manoeuvre)

Table 6: Probability of collision for real scenarios, *closeap* results

Several geometries were tested to find the limitations in the collision model used in *closeap*. The efforts are focused to find specific cases where the mathematical models used in *closeap* face singularities. Three categories of collisions face the model singularities.

Conjunction plane is defined using the relative velocity and relative position vectors. In the general form in *closeap*, it is defined as follows:

\[
\vec{i} = \frac{\vec{v}_x}{v_x} \quad \vec{j} = -\frac{\vec{v}_y}{|v_y|} \quad \vec{k} = \vec{i} \times \vec{j}
\]

And if the relative position (range or miss distance) is zero, then the plane uses the x-direction of the inertial frame as a reference, and:
Keeping this definition in mind, and remembering the general formula to estimate the collision probability, the limiting cases are:

1- Zero relative velocity: These categories of orbits will have the state vector of target and chaser as follows:

\[
X_{\text{target}} = \begin{bmatrix} x \\ y \\ z \\ \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{v}_z \end{bmatrix}, \quad X_{\text{chaser}} = \begin{bmatrix} x' \\ y' \\ z' \\ \mathbf{v}_x' \\ \mathbf{v}_y' \\ \mathbf{v}_z' \end{bmatrix}
\]

2- Geometries that lead to a zero-determinant projected position covariance matrix (figures 15 and 16). Remembering the collision plane defined by relative position and velocity vectors in J2000 coordinates, the transformation matrix and the projected covariance in the collision plane will be:

\[
C_{\text{J2000-bplane}} = \begin{bmatrix} i \\ j \\ k \end{bmatrix}, \quad F = C_{\text{J2000-bplane}} \times (F_t + F_c), \quad P' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times [F] \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

That gives a 2*2 projected covariance matrix on the collision plane. With this definition of transformation matrix, the target and chaser state vectors that lead to this condition will be:

\[
X_{\text{target}} = \begin{bmatrix} x \\ y \\ z \\ \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{v}_z \end{bmatrix}, \quad X_{\text{chaser}} = \begin{bmatrix} x \\ y \\ z \\ \mathbf{v}_x + \Delta \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{v}_z \end{bmatrix}
\]

3- Parallel relative position and relative velocity vectors (figure 17)

CONCLUSION

The result of the tests with closeap has shown an acceptable assessment of different conjunctions with different geometries. But the covariance has a significant role in more accurate estimations. Low accuracy of TLE data is one of the main factors in difficulties regarding the conjunction prediction.

The tests also showed how the estimated probability changes with geometry of conjunction, defined by miss distance, azimuth, elevation, relative velocity, approach azimuth and approach elevation. The angles for minimum probabilities estimated, correspond to geometries whose corresponding probability may decrease below the threshold with an increase in the covariance.

Also, there are limiting scenarios for the current model with the specific encounter plane definition. These cases include geometries with zero relative velocity, and parallel miss distance and relative velocity vector, along with the cases where projection of covariance on the encounter plane leads to zero determinant.

REFERENCES

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3- Relating Position Uncertainty to Maximum Conjunction Probability, Salvatore Alfano, AAS 03-548
5- CNES operational experiences in collision avoidance for LEO satellites, Francois Laporte, Monique Moury, Xavier Pena, ISU Symposium, February 2009.
6- Space Debris, Models and Risk Analysis, H.Klinkrad, Springer, 2006
TABLES AND FIGURES

### Table 1: TLE errors, orbital eccentricity lower than 0.1

<table>
<thead>
<tr>
<th>$h_p$</th>
<th>RAD</th>
<th>A-T</th>
<th>C-T</th>
<th>A-T</th>
<th>C-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 800 km</td>
<td>183.7</td>
<td>360.5</td>
<td>266.8</td>
<td>244.0</td>
<td>137.2</td>
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<td>800 km ≤ $h_p$ ≤ 2500 km</td>
<td>318.3</td>
<td>250.6</td>
<td>530.6</td>
<td>307.9</td>
<td>139.4</td>
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<tr>
<td>≥ 25000 km</td>
<td>864.4</td>
<td>790.2</td>
<td>407.3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 2: TLE errors, orbital eccentricity higher than 0.1

<table>
<thead>
<tr>
<th>$h_p$</th>
<th>RAD</th>
<th>A-T</th>
<th>C-T</th>
<th>A-T</th>
<th>C-T</th>
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<tr>
<td>&lt; 800 km</td>
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<td>5891.1</td>
<td>670.2</td>
<td>2859.5</td>
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<td>2017.8</td>
<td>2663.3</td>
<td>1329.9</td>
<td>2333.6</td>
<td>1456.1</td>
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<tr>
<td>≥ 250000 km</td>
<td>1115.7</td>
<td>778.3</td>
<td>214.0</td>
<td>-</td>
<td>-</td>
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</table>

### Table 4: Test orbits in J2000 and Keplerian system

<table>
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<tr>
<th>X (km)</th>
<th>Y(km)</th>
<th>Z(km)</th>
<th>Vx(km/s)</th>
<th>Vy(km/s)</th>
<th>Vz(km/s)</th>
<th>semi-major axis(km)</th>
<th>Eccentricity</th>
<th>Inclination (deg)</th>
<th>Asc.Node (deg)</th>
<th>Arg. Perigee(deg)</th>
<th>Tr.Anomaly(deg)</th>
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<td>-180:180 &amp; -180:180</td>
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<td>0,000000</td>
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</tbody>
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### Table 5: Test cases for figures 11 and 12 and 13

<table>
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<tr>
<th>test</th>
<th>R (km)</th>
<th>range rate</th>
<th>$h_{\text{app}}$(deg)</th>
<th>$A_{\text{app}}$(deg)</th>
<th>h(deg)</th>
<th>A (deg)</th>
</tr>
</thead>
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<td>1</td>
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<td>0.01</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>2</td>
<td>0.5</td>
<td>0.01</td>
<td>0</td>
<td>-180:180</td>
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<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.01</td>
<td>30</td>
<td>-180:180</td>
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<td>0</td>
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<tr>
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<td>0.01</td>
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<td>30</td>
</tr>
<tr>
<td>A</td>
<td>changing</td>
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<td>0</td>
<td>-90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
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<td>-90</td>
<td>0</td>
<td>0</td>
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<tr>
<td>C</td>
<td>changing</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-90</td>
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</table>
Figure 6 - Probability and approach azimuth in LEO

Figure 7 - Probability and approach elevation in LEO

Figure 8 - Probability and Approach Elevation, the model is limits for 90 degree angle

Figure 9 - Probability and approach azimuth in GEO

Figure 10 - Probability and approach azimuth in MEO
Figure 11 - Probability of Collision in LEO vs. miss distance. Test cases A, B, C are defined in table 5

Figure 12 - Maximum probability is achieved when miss distance and approach velocity are in the same direction

Figure 13 - Probability of Collision decreases with miss distance. The probability decreases significantly for test 2 of table and at 1.0 km miss distance the probability reaches 1.62e-20

Figure 14 - Probability of Collision change with covariance
Figure 15 - 2\textsuperscript{nd} category of limiting conjunctions with a zero-determinant projected position covariance matrix, for target and chaser in LEO

Figure 16 - 2\textsuperscript{nd} category of limiting conjunctions with a zero-determinant projected position covariance matrix, for target and chaser in LEO
Figure 17 - 3rd category of limiting conjunction with parallel miss distance and relative velocity, Target and Chaser in GEO